Power system dynamic stability and voltage regulation enhancement using an optimal gain vector

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Abstract

This paper addresses a control methodology to enhance power system dynamic stability and voltage regulation by augmenting existing generator controls (conventional PSS and AVR) using an optimal static gain vector. The control design problem is reduced to finding a new control loop including a simple fixed gain vector. In order to optimal tune gain elements, the problem is formulated via an $H_\infty$ static output feedback ($H_\infty$-SOF) control technique, and the solution is easily found using an iterative linear matrix inequalities (ILMI) algorithm. Real-time experiments have been performed for a longitudinal four-machine infinite-bus system on the Analog Power System Simulator at the Research Laboratory of the Kyushu Electric Power Co. The proposed robust technique is shown to maintain the robust performance and minimize the effects of disturbances.

Keywords: Power system stabilizer; Voltage regulation; $H_\infty$ control; Static output feedback; LMI

1. Introduction

Power systems continuously experience changes in operating conditions due to variations in generation/load and a wide range of disturbances. Power system stability and voltage regulation have been considered an important problem for secure system operation over many years (Kundur et al., 2004). Currently, because of expanding physical setups, functionality and complexity of power systems, the mentioned problems become more significant than in the past. That is why in recent years a great deal of attention has been paid to application of advanced control techniques to power systems.

Conventionally, the automatic voltage regulation and power system stabilizer (AVR–PSS) design is considered as a sequential design including two separate stages. Firstly, the AVR is designed to meet the specified voltage regulation performance and then the PSS is designed to satisfy the stability and required damping performance.

It is known that stability and voltage regulation are ascribed to different model descriptions, and it has been long recognized that AVR and PSS have inherent conflicting objectives (Law, Hill, & Godfrey, 1994a, 1994b; Venikov & Stroev, 1971).

In the last two decades, some studies have considered an integrated design approach to AVR and PSS design using domain partitioning (Venikov & Stroev, 1971), robust pole-replacement (Soliman & Sakar, 1988) and adaptive control (Malik, Hope, Gorski, Uskakov, & Rackevich, 1986). Moreover, several control methods have recently been made to coordinate the various requirements for stabilization and voltage regulation within the one new control structure (Bevrani & Hiyama, 2006; Guo, Hill, & Wang, 2001; Heniche, Bourles, & Houry, 1995; Wang & Hill, 1996; Yadaiah, Kumar, & Bhattacharya, 2004).

Although most of these approaches have been proposed based on new contributions in modern control systems, they are not well suited to meet the design objectives in a real multi-machine power system because of following two main reasons: (i) The complexity of control structure, numerous unknown design parameters and neglecting real
constraints can be frequently seen in the most of new suggested controllers. While in real world power systems, usually controllers with simple structure are desirable. That is why electric industry still uses the simple PI, PID and Lead-lag controllers that their parameters are commonly tuned based on classical, experiences and trial-and-error approaches. (ii) Experience shows that although the conventional PSS and AVR systems are incapable of obtaining good dynamical performance for a wide range of operating conditions and disturbances, the electric industry is too conservative to open the conventional control loops and test the novel/advanced controllers because of some probable risks, bugs and/or having a complex structure.

In response to above problems, this paper presents a methodology to enhance the stability and voltage regulation of existing real power system without opening their conventional control devices. The methodology provides a simple gain vector in parallel with the conventional PSS and AVR devices. The methodology to enhance the stability and voltage regulation of existing real power system without opening their conventional control loops and test the novel/advanced controllers because of some probable risks, bugs and/or having a complex structure.

To demonstrate the efficiency of the proposed control method, some real time nonlinear laboratory tests have been performed on a four-machine infinite-bus system using the large scale Analog Power System Simulator at the Research Laboratory of the Kyushu Electric Power Company (Japan). The obtained results are compared with a conventional AVR–PSS system.

2. Proposed control strategy

2.1. A background on \( H_\infty \)-SOF control design

This section gives a brief overview for the \( H_\infty \)-SOF control design. Consider a linear time invariant system \( G(s) \) with the following state-space realization.

\[
\dot{x}_i = A_ix_i + B_1iw_i + B_2iu_i,
\]

\[
G_i(s) : z_i = C_{i1}x_i + D_{12}u_i,
\]

\[
y_i = C_{2i}x_i,
\]

where \( x_i \) is the state variable vector, \( w_i \) is the disturbance and area interface vector, \( u_i \) is the control input vector, \( z_i \) is the controlled output vector and \( y_i \) is the measured output vector. The \( A_i, B_1i, B_2i, C_{1i}, C_{2i} \) and \( D_{12i} \) are known real matrices of appropriate dimensions.

The \( H_\infty \)-SOF control problem for the linear time invariant system \( G_i(s) \) with the state-space realization of (1) is to find a gain matrix \( K_i \) (\( u_i = K_iy_i \)), such that the resulted closed-loop system is internally stable, and the \( H_\infty \) norm from \( w_i \) to \( z_i \) (Fig. 1) is smaller than \( \gamma \), a specified positive number, i.e.

\[
\| T_{z_i,w_i} \|_{\infty} < \gamma.
\]

It is notable that the \( H_\infty \)-SOF control problem can be transferred to a generalized SOF stabilization problem which is expressed via the following theorem (Cao, Lam, Sun, & Mao, 1998).

**Theorem.** The system \((A, B, C)\) is stabilizable via SOF if and only if there exist \( P \succ 0, X \succ 0 \) and \( K_i \) satisfying the following quadratic matrix inequality

\[
\begin{bmatrix}
A^TX + XA - PB_1^TP - PB_2^TP + PBB_1^TP + PBB_2^TP & (B_1^TX + K_iC_i)^T \\
B_1^TX + K_iC_i & -I
\end{bmatrix} < 0.
\]

Here, the matrices \( A, B \) and \( C \) are constant and have appropriate dimensions. The \( X \) and \( P \) are symmetric and positive-definite matrices.

Since a solution for the consequent non convex optimization problem (3) cannot be directly achieved by using general and convex LMI techniques (Bevrani & Hiyama, 2007; Boyd, El Chaoui, Feron, & Balakrishnan, 1994; Swarnakar, Marquez, & Chen, 2007), a variety of methods were proposed by many researchers with many analytical and numerical methods to approach a local/global solution. In this paper, to solve the resulted SOF problem, an iterative LMI is used based on the existence necessary and sufficient condition for SOF stabilization, via the \( H_\infty \) control technique.

Using above theorem and the bounded real lemma (Zhou, Doyle, & Glover, 1996), the \( K_i \) is an \( H_\infty \)-SOF controller for system (1), if and only if there exists \( X \succ 0 \) such that

\[
\bar{X} \bar{B}_iK_i\bar{C}_i + (\bar{X} \bar{B}_iK_i\bar{C}_i)^T + \bar{A}_i^T\bar{X} + \bar{X}\bar{A}_i < 0,
\]

Fig. 1. Closed-loop system via \( H_\infty \)-SOF control.
where

\[ \mathbf{X} = \begin{bmatrix} X & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} A_i & B_{li} & 0 \\ 0 & -\gamma I/2 & 0 \\ C_{li} & 0 & -\gamma I/2 \end{bmatrix}, \]

\[ \mathbf{B}_i = \begin{bmatrix} B_{2i} \\ 0 \\ D_{12i} \end{bmatrix}, \quad \mathbf{C}_i = [C_{2i} \ 0 \ 0]. \] (5)

Here, \( \mathbf{A}_i, \mathbf{B}_i \) and \( \mathbf{C}_i \) are three generalized matrices.

### 2.2. Modeling

In order to design a robust power system controller, it is first necessary to consider an appropriate linear mathematical description of multi-machine power system with two-axis generator models. In the viewpoint of “generator unit”, the nonlinear state space representation model for such a system has the form

\[ \dot{x}_{gi} = f(x_{gi}, u_{gi}), \] (6)

where the states

\[ x_{gi}^T = [x_{1gi} \ x_{2gi} \ x_{3gi} \ x_{4gi}] = [\delta_i \ \omega_i \ E_{gi}^q \ E_{gi}^d] \] (7)

are defined as deviation form the equilibrium values

\[ x_{gi}^T = [\delta_{i}^{e} \ \omega_{i}^{e} \ E_{gi}^{e q} \ E_{gi}^{e d}]. \] (8)

Using the linearization technique and after some manipulation, the nonlinear state Eqs. (6) can be expressed in the form of following linear state space model,

\[ \dot{x}_i = A_i x_i + B_i u_i, \] (9)

where

\[ A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & \frac{D_i}{M_i} & a_{23} & a_{24} \\ a_{31} & 0 & a_{33} & \frac{G_{pi} \Delta x_{di}}{T_{d0i}} \\ a_{41} & 0 & \frac{G_{qi} \Delta x_{qi}}{T_{q0i}} & a_{44} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \] (10)

and

\[ \Delta x_{di} = x_{di} - x_{di}^e, \quad \Delta x_{qi} = x_{qi} - x_{qi}^e. \] (11)

The \( a_{kj} \) elements are described in Appendix. The other parameters are defined as follows: \( \delta_i \), Machine rotor angle; \( \omega_i \), Machine rotor speed; \( E_{gi}^q \), \( q \)-axis internal machine voltage; \( E_{gi}^d \), \( d \)-axis internal machine voltage; \( D_i \), Damping constant; \( M_i \), Inertia constant; \( G_{pi} \), Driving point conductance; \( T_{d0i} \), \( d \)-axis open circuit transient time constant; \( T_{q0i} \), \( q \)-axis open circuit transient time constant; \( x_{di} \), \( d \)-axis synchronous reactance; \( x_{qi} \), \( q \)-axis synchronous reactance.

Considering the conventional AVR–PSS system, the overall system control input can be written in the following form.

\[ u_{gi} = u_{ci} + u_i, \] (12)

where, \( u_{ci} \) is the output of conventional AVR–PSS system and the \( u_i \) is the new control input (Fig. 2). Therefore, the overall system can be described as follows:

\[ \dot{x}_i = A_i x_i + B_i u_i \] (13)

and

\[ x_i^T = [x_{gi1} \ x_{gi2} \ x_{gi3} \ x_{gi4}]_i \times (4+m). \] (14)

Here, the \( x_{ci} \) shows the state vector of conventional AVR–PSS system and \( m \) represents its dynamic order.

### 2.3. Proposed control framework

The overall control structure using SOF control design for an assumed power system is shown in Fig. 2, where blocks PSS and AVR represents the existing conventional power system stabilizer and voltage regulators. Here, the electrical power signal \( \Delta p_{ei} \) is considered as input signal for the PSS unit. The optimal gain vector (OGV) uses the terminal voltage \( \Delta v_{ti} \), electrical power \( \Delta p_{ei} \) and machine speed \( \Delta \omega_i \) as input signals. The \( \Delta v_{refi} \) and \( d_i \) show the reference voltage deviation and system disturbance input, respectively.

Using the linearized model for power system “i” in the form of (1) and performing the standard \( H_{\infty} \)-SOF configuration with considering appropriate controlled output signals results an effective control framework.

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**Fig. 2.** Overall control structure.

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**Fig. 3.** The proposed \( H_{\infty} \)-SOF control framework.
which is shown in Fig. 3. This control structure adapts the H\(_\infty\)–SOF control technique with the described power system control targets and allows a direct trade-off between voltage regulation and closed-loop stability by merely tuning of a vector gain.

Here, disturbance input vector \(w_i\) controlled output vector \(z_i\) and measured output vector \(y_i\) are considered as follows:

\[
w_i^T = [\Delta v_{refi} \; d_i],
\]

\[
z_i^T = [\eta_1, \Delta v_{refi} \; \eta_2, \Delta \delta, \eta_3, u_i],
\]

\[
y_i^T = [\Delta v_{refi} \; \Delta p_{el}, \Delta \gamma_0].
\]

The \(\Delta v_{refi}\) and \(\Delta p_{el}\) can be easily expressed via the specified system states, and the \(\eta_1, \eta_2\) and \(\eta_3\) are constant weights that must be chosen by the designer to achieve a desired closed-loop performance. Since the vector \(z_i\) properly covers all significant controlled signals which must be minimized by an ideal AVR–PSS design, it is expected that the proposed robust controller should be able to satisfy the voltage regulation and stabilizing objectives, simultaneously. It is notable that, since the solution must be obtained through the minimizing of an H\(_\infty\) optimization problem, the designed feedback system satisfies the robust stability and voltage regulation performance for the overall closed-loop system. Moreover, the developed ILMI algorithm (which is described in the next section) provides an effective and flexible tool for finding an appropriate solution in the form of a simple static gain controller.

\[
\begin{bmatrix}
\dot{X}_i + A_i^T X_i + \bar{X}_i A_i - P_B B_i^T X_i - \bar{X}_i B_i P_i + P_B B_i^T P_i - a_i \bar{X}_i (B_i^T X_i + K_i C_i)^T - I \\
B_i^T X_i + K_i C_i
\end{bmatrix} < 0,
\]

\[
\dot{X}_i > 0.
\]

\[u_i = K_i y_i, \quad K_i \in K_{sof},\]

such that

\[
\| T_{zow}(s) \|_\infty < \gamma^*, \quad |\gamma - \gamma^*| < \epsilon,
\]

where \(\epsilon\) is a small positive number. The performance index \(\gamma^*\) indicates a lower bound such that the closed-loop system is H\(_\infty\) stabilizable. The optimal performance index (\(\gamma\)), can be obtained from the application of a full dynamic H\(_\infty\) dynamic output feedback control method. The proposed algorithm, which gives an iterative LMI solution for above optimization problem includes the following steps:

**Step 1.** Set initial values and compute the generalized system (\(A_i, B_i, C_i\)) as shown in (5), for the given power system including conventional AVR–PSS system. For this purpose, according to (13), the matrix \(A_i\) has the following form and the elements of other matrices in (5) can be obtained based on the structure of the used excitation system and AVR–PSS unit.

\[
A_i = \begin{bmatrix}
A_{\Xi i} & 0_{(4 \times m_n)} \\
0_{(m_n \times 4)} & A_{\Xi_{o i}}
\end{bmatrix},
\]

The “\(\epsilon\)” is used for the conventional AVR–PSS system.

**Step 2.** Set \(i = 1, \Delta \gamma = \Delta \gamma_0\) and let \(\gamma_i = \gamma_0 > \gamma\). \(\Delta \gamma_0\) and \(\gamma_0\) are positive real numbers.

**Step 3.** Select \(Q > 0\), and solve \(\bar{X}\) from the following algebraic Riccati equation

\[
A_i^T \bar{X} + \bar{X} A_i - \bar{X} B_i B_i^T \bar{X} + Q = 0.
\]

Set \(P_1 = \bar{X}\).

**Step 4.** Solve the following optimization problem for \(\tilde{X}_i, K_i\) and \(a_i\).

Minimize \(a_i\) subject to the LMI constraints:

2.4. **ILMI algorithm**

It is well-known that static output feedback stabilization is still an open problem. Its reformulation generally leads to bilinear matrix inequalities (BMI) which are non-convex. This kind of problem is usually solved by an iterative algorithm that may not converge to an optimal solution.

Here, in order to solve the H\(_\infty\)-SOF, an iterative LMI algorithm has been used. The algorithm is mainly based on the given idea by Cao et al. (1998). The key point is to formulate the H\(_\infty\) problem via a generalized static output stabilization feedback such that all eigenvalues of \((A - B K_i C)\) shift towards the left half plane in the complex s-plane, to close to feasibility of (3). The described theorem in the previous section gives a family of internally stabilizing SOF gains is defined as \(K_{sof}\). The desirable solution \(K_i\) is an admissible SOF law

\[u_i = K_i y_i, \quad K_i \in K_{sof},\]
The proposed iterative LMI algorithm, which is summarized in the flowchart of Fig. 4, shows that if one simply perturbs \( \bar{A}_i \) to \( \bar{A}_i - (a/2)I \) for some \( a > 0 \), a solution of the matrix inequality (3) can be obtained for the performed generalized plant. That is, there exist a real number \( a > 0 \) and a matrix \( P > 0 \) to satisfy inequality (22). Consequently, the closed-loop system matrix \( \bar{A}_i - \bar{B}_iK\bar{C}_i \) has eigenvalues on the left-hand side of the line \( \Re(s) = a \) in the complex s-plane. Based on the idea that all eigenvalues of \( \bar{A}_i - \bar{B}_iK\bar{C}_i \) are shifted progressively towards the left half plane through the reduction of \( a \). The given generalized eigenvalue minimization in the proposed iterative LMI algorithm guarantees this progressive reduction.

2.5. Weights selection

The vector \( \eta_i = [\eta_{1i}, \eta_{2i}, \eta_{3i}] \) is a constant weight vector that must be chosen by the designer to get the desired closed-loop performance. The selection of these weights is dependent on specified voltage regulation and damping performance goals. In fact an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives. It is notable that \( \eta_{3i} \) sets a limit on the allowed control signal to penalize fast changes, large overshoot with a reasonable control gain to meet the physical constraints. Therefore, the selection of constant weights entails a compromise among several performance requirements.

One can simply fix the weights to unity and use the method with regional pole placement technique for performance tuning (Gahinet & Chilali, 1996). Here, for the sake of weight selection, the following steps are simply considered through the proposed ILMI algorithm:

**Step 1.** Set initial values, e.g. [1 1 1].

**Step 2.** Run the ILMI algorithm (summarized in Fig. 4).

**Step 3.** If the ILMI algorithm gives a feasible solution such that satisfies the robust \( H_\infty \) performance and the gain constraint; the assigned weights vector is acceptable. Otherwise retune \( \eta_i \) and go to Step 2.

3. Real time implementation

To illustrate the effectiveness of the proposed control strategy, a real time experiment has been performed on the large scale Analog Power System Simulator at the Research Laboratory of the Kyushu Electric Power Company. For the purpose of this study, a longitudinal four-machine infinite bus system is considered as a test system. A single line representation of the study system is shown in Fig. 5. Although, in the given model the number

![Fig. 4. Iterative LMI algorithm.](image)

![Fig. 5. Four-machine infinite-bus power system.](image)
of generators is reduced to four, it closely represents the dynamic behavior of the west part of Japan (West Japan Power System), and it is widely used by researchers (Bevrani & Hiyama, 2007; Hiyama, Oniki, & Nagashima, 1996; Hiyama, Kawakita, & Ono, 2004; Hiyama, Kojima, Ohtsu, & Furukawa, 2005). The most important global and local oscillation modes of actual system are included. For the study system, the local mode for each corresponding unit, and the low frequency global mode are around 1.5 Hz and 0.3 Hz, respectively. Each unit is a thermal unit, and has a separately conventional excitation control system as shown in Figs. 6a and b.

Each unit has a full set of governor-turbine system (governor, steam valve servo-system, high-pressure turbine, intermediate-pressure turbine, and low-pressure turbine) which is shown in Fig. 7. The generators, lines, conventional excitation system and governor-turbine parameters are given in Tables 1–4, respectively.

Unit 1 is selected to be equipped with robust control, and therefore our objective is to apply the control strategy described in the previous section to controller design for unit 1. The whole power system has been implemented in the mentioned laboratory. Fig. 8 shows the overview of the applied laboratory experiment devices including the

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**Table 1**

<table>
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<th>Unit no.</th>
<th>( M_i ) (s)</th>
<th>( D_i )</th>
<th>( x_{di} ) (pu)</th>
<th>( x_{0i} ) (pu)</th>
<th>( x_{qi} ) (pu)</th>
<th>( T_{0di} ) (s)</th>
<th>( T_{0qi} ) (s)</th>
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<td>0.0873</td>
<td>1000</td>
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**Table 3**

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**Table 4**

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<tr>
<td>( K_3 ) (pu)</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>( M_1 ) (pu/Min)</td>
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<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
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<tr>
<td>( M_2 ) (pu/Min)</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
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<tr>
<td>( M_3 ) (pu/Min)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
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<tr>
<td>( N_1 ) (pu/Min)</td>
<td>-0.50</td>
<td>-0.50</td>
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<td>( N_2 ) (pu/Min)</td>
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<td>( N_3 ) (pu/Min)</td>
<td>-0.50</td>
<td>-0.50</td>
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<td>-0.50</td>
</tr>
</tbody>
</table>

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Fig. 6. Conventional excitation control system: (a) for units 2 and 3 and (b) for units 1 and 4.

Fig. 7. (a) Conventional speed governing system and (b) detailed turbine system.
control/monitoring desks. A digital oscilloscope and a notebook computer (shown in Fig. 8b) are used for monitoring purposes.

The proposed control loop (Fig. 9) has been built in a personal computer where connected to the power system using a digital signal processing (DSP) board equipped with analog to digital (A/D) and digital to analog (D/A) converters as the physical interfaces between the personal computer and the analog power system hardware. In Fig. 9, the input/output scaling blocks are used to match the PC based controller and the Analog Power System hardware, signally. High frequency noises are removed by appropriate low pass filters.

Then, applying the proposed $H_{\infty}$-SOF control methodology an OGV for the problem at hand is obtained as follows:

$$K_{1, \text{SOF}} = [9.5899 \ 7.8648 \ 1.2990].$$

The considered constraints on limiters and control loop gains are set according to the real power system control units and close to ones that exist for the conventional AVR–PSS units. The used constant weight vector ($\eta$) is given in Appendix.

4. Experiment results

The performance of the closed-loop system using the proposed OGV in comparison of a pure conventional AVR–PSS system is tested in the presence of voltage deviation, faults and system disturbance. The configuration of the applied conventional power system stabilizer, which was accurately tuned by the system operators, is illustrated in Fig. 10. The conventional PSS parameters are listed in Table 5.

During the first test scenario, the output setting of unit 1 is fixed to 0.3 pu. Fig. 11 shows the electrical power, terminal voltage and machine speed of unit 1, following a fault on the line between buses 11 and 12 at 2 s. To force a more critical situation, the faulted line is isolated from the network just after four cycles from the fault. It can be seen that the system response is quite improved using the designed feedback gains.

Furthermore, the size of resulted stable region by the proposed method is significantly enlarged in comparison of conventional AVR–PSS controller. To show this fact, the critical power output from unit 1 in the presence of a three-phase to ground fault is considered as a good measure. To investigate the critical point, the real power output of unit 1 is increased from 0.3 pu (The setting of the real power output from the other units is fixed at the values shown in Fig. 5). Using the conventional AVR–PSS structure, the resulted critical power output from unit 1 to be 0.31 pu.

![Fig. 8. Performed laboratory experiment: (a) overview of analog power system simulator and (b) the control/monitoring desks.](image)

![Fig. 9. The performed computer based control loop.](image)

![Fig. 10. Conventional power system stabilizer.](image)

Table 5

<table>
<thead>
<tr>
<th>Conventional PSS parameters</th>
<th>$T_1$ (s)</th>
<th>$G_{PSS}$</th>
<th>$U_{\text{max}}$ (pu)</th>
<th>$T_1$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$ (s)</td>
<td>5.00</td>
<td>10.00</td>
<td>1.00</td>
<td>0.025</td>
</tr>
<tr>
<td>$T_2$ (s)</td>
<td>0.056</td>
<td>0.054</td>
<td>0.037</td>
<td>0.53</td>
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</table>
and in case of tight tuning of CPSS parameters it could not be higher than 0.52 pu. For the proposed control method, the critical power output, as shown in Table 6, is increased to 0.94 pu. The system response for a fault between buses 11 and 12, while the output setting of unit 1 is increased to 0.7 pu is shown in Fig. 12.

In the second test case, the performance of designed controllers was evaluated in the presence of a 0.05 pu step disturbance injected at the voltage reference input of unit 1 at 20 s. Fig. 13 shows the closed-loop response of the power systems fitted with the conventional control and the proposed robust control design. Better performance is achieved by the proposed control strategy. In the next scenario, the closed-loop system response is examined in the face of a step disturbance ($d_t$) at 20 s. The result is shown in Fig. 14. Comparing the experiment results shows that the robust design achieves robustness against the voltage deviation, disturbance and line fault with a quite good voltage regulation and damping performance.

Finally, to demonstrate the simultaneous damping of local (fast) and global (slow) oscillation modes, filtering analysis has been performed. The laboratory results for the speed deviation of unit 1, following a fault on the line between buses 11 and 12 are shown in Fig. 15 (in this experiment, the fault was happen at 2 s and the output setting of unit 1 was fixed at 0.45 pu).

Table 6

<table>
<thead>
<tr>
<th>Control design</th>
<th>Critical power output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed design</td>
<td>0.94 (pu)</td>
</tr>
<tr>
<td>Conventional AVR–PSS</td>
<td>0.52 (pu)</td>
</tr>
</tbody>
</table>

Fig. 11. System response for a fault between buses 11 and 12. Solid (using OGV), dotted (conventional AVR–PSS).

Fig. 12. System response for a fault between buses 11 and 12, while the output setting of unit 1 is fixed to 0.5 pu. Solid (using OGV), dotted (conventional AVR–PSS).
Fig. 13. System response for a 0.05 pu step change at the voltage reference input of unit 1. Solid (using OGV), dotted (conventional AVR–PSS).

Fig. 14. System response for a step disturbance at 20 s. Solid (using OGV), dotted (conventional AVR–PSS).
5. Conclusion

In order to achieve simultaneous enhancement of power system stability and voltage regulation, a new control strategy is developed using an $H_{\infty}$-SOF control technique and a developed iterative LMI algorithm. The proposed method was applied to a four-machine infinite bus power system, through a laboratory real-time experiment, and the results are compared with a conventional AVR–PSS design. The performance of the resulting closed-loop system is shown to be satisfactory over a wide range of operating conditions.

As shown in the nonlinear real-time simulation results, the proposed coordination through a new optimal feedback loop has brought a significant improvement to power system performance and has increased the stable region of operation. The resulting controller is not only robust but it also allows direct effective trade-off between voltage regulation and damping performance. Furthermore, because of simplicity of structure, decentralized property, ease of formulation and flexibility, the design methodology can be practically implemented.

Acknowledgments

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Appendix

The nonlinear model of (6) can be presented as follows:

\begin{align*}
\dot{x}_{1gi} &= x_{2gi}, \\
\dot{x}_{2gi} &= -\left(\frac{D_i}{M_i} + \frac{1}{M_i}\right)x_{2gi} - \frac{1}{M_i}\Delta P_{ci}(x), \\
\dot{x}_{3gi} &= -\left(\frac{1}{T_{di}}\right)x_{3gi} - \left(\frac{\Delta x_{di}(x)}{T_{di}}\right)\Delta I_{gi}(x) + u_{qi}, \\
\dot{x}_{4gi} &= -\left(\frac{1}{T_{qi}}\right)x_{4gi} - \left(\frac{\Delta x_{qi}(x)}{T_{qi}}\right)\Delta I_{qi}(x),
\end{align*}

where

\begin{align*}
\Delta P_{ci}(x) &= (E_{di}I_{di} + E_{qi}I_{qi}) - (E'_{di}I'_{di} + E'_{qi}I'_{qi}), \\
I_{di} &= \sum_k [G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}]E'_{dk} + \sum_k [G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}]E'_{qk}, \\
I_{qi} &= \sum_k [B_{ik} \cos \delta_{ik} - G_{ik} \sin \delta_{ik}]E'_{dk} + \sum_k [G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}]E'_{qk}.
\end{align*}

A detailed description of all symbols and quantities can be found in Sauer and Pai (1998). The elements of $A_{gi}$ matrix in (10) are

\begin{align*}
a_{21} &= -\frac{1}{M_i} \frac{\partial f_{j1}(x)}{\partial x_{1gi}}, \\
a_{23} &= \frac{\left[G_{qi}E'_{qi} - B_{qi}E'_{qj} + I'_{qi}\right]}{M_i} - \frac{1}{M_i} \frac{\partial f_{j1}(x)}{\partial x_{3gi}}.
\end{align*}
\[ a_{24} = -\left[\frac{G_i E_{ii}^c + B_f E_{iq}^c + F_{ik}^c}{M_i} \right] \frac{\partial f_{1i}(x)}{\partial x_{4qi}} \bigg|_{x_{us}}, \]
\[ a_{31} = -\left(\frac{\partial^2 f_{1i}(x)}{\partial x_{1qi}^2} \right) \bigg|_{x_{us}}, \]
\[ a_{33} = -\frac{1}{T_{d0}} + \frac{B_i \Delta x_{di}}{T_{d0}}, \]
\[ a_{41} = -\left(\frac{\partial^2 f_{1i}(x)}{\partial x_{1qi}^2} \right) \bigg|_{x_{us}}, \]
\[ a_{44} = -\frac{1}{T_{q0}} + \frac{B_i \Delta x_{qi}}{T_{q0}}, \]

where

\[ f_{1i}(x) = x_{4qi} \Delta I_{di}(x) + x_{3qi} \Delta I_{qi}(x) + \sum_{k \neq i} \left( [E_{iq}^c \eta_{ik}(\delta) + E_{qi}^c \tilde{q}_{ik}(\delta)] x_{4qk} + [E_{iq}^c \bar{v}_{ik}(\delta) + E_{qi}^c \tilde{v}_{ik}(\delta)] x_{3qk} + [E_{id}^c \bar{v}_{ik}(\delta) + E_{qi}^c \tilde{v}_{ik}(\delta)] \sin \phi_{ik} \right), \]

\[ f_{2i}(x) = \sum_{k \neq i} \left[ \bar{h}_{ik}(\delta) x_{4qk} + \bar{v}_{ik}(\delta) x_{3qk} + \bar{\tilde{v}}_{ik}(\delta) \sin \phi_{ik} \right], \]

\[ f_{3i}(x) = \sum_{k \neq i} \left[ \tilde{h}_{ik}(\delta) x_{4qk} + \tilde{v}_{ik}(\delta) x_{3qk} + \tilde{\tilde{v}}_{ik}(\delta) \sin \phi_{ik} \right], \]

\[ \eta_{ik}(\delta) = G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}, \]

\[ \bar{h}_{ik}(\delta) = B_{ik} \cos \delta_{ik} - G_{ik} \sin \delta_{ik}, \]

\[ v_{ik}(\delta) = G_{ik} \sin \delta_{ik} + B_{ik} \cos \delta_{ik}, \]

\[ \tilde{v}_{ik}(\delta) = B_{ik} \sin \delta_{ik} - G_{ik} \cos \delta_{ik}, \]

\[ v_{1i}(\delta) = 2g_{1i} \sin \frac{\delta_{ik} + \delta_{ik}}{2} + 2g_{2i} \cos \frac{\delta_{ik} + \delta_{ik}}{2}, \]

\[ \phi_{ik} = 0.5(x_{1qi} - x_{3qk}), \]

\[ \bar{v}_{ik}(\delta) = 2g_{2i} \sin \frac{\delta_{ik} + \delta_{ik}}{2} - 2g_{1i} \cos \frac{\delta_{ik} + \delta_{ik}}{2}, \]

\[ \delta_{ik} = \delta_i - \delta_k, \]

\[ g_{1i} = G_{ik} E_{ik}^c - B_{ik} E_{qk}^c, \]

\[ g_{2i} = G_{ik} E_{ik}^c + B_{ik} E_{qk}^c. \]

Constant weights : \( \eta_1 = [0.25 \ 0.1 \ 5]. \)

**References**


